Elburton Primary School

Multiplication & Subtraction
Models & Representations Policy

October 2022	Policy Agreed
July 2024	Policy Review Date
Date	**Description**

This progression in calculations policy has been devised using the guidance from White Rose Maths Hub as well as the NCETM to support understanding in the expectations for fluency of the National Curriculum in England (2014) as well as the progression of calculation concepts through a child's mathematical development. Teaching staff have adapted and agreed on the guidance given in order to make it bespoke to the curriculum here at Elburton Primary School.

Principles

- To support children's development and understanding of calculations through the use of concrete, pictorial and abstract methods.
- To support children to develop a deeper understanding of number as well as mental and written calculation.
- To develop, using a Mastery Maths approach, number awareness and fluency, which is supported through the use of models and images.
- To ultimately develop proficiency with the expected formal written methods by the end of Year 6.
- Specific practical resources and models have been suggested as well as the benefits of these for manipulative to support children in developing the conceptual understanding that will enable them to move efficiently towards the formal written methods expected.
- It is expected that all children will work towards the fluency goals for each age group but that, where necessary, teachers will use approaches and materials from NCETM, White Rose Maths and earlier year groups to bridge any gaps in a child's understanding.
- Teachers should have an understanding of the expectations and progression for all year groups, regardless of which year group they teach.
- All teachers have progression maps linked to the objectives set out in the National Curriculum 2014.

Concrete-Pictorial-Abstract Representations

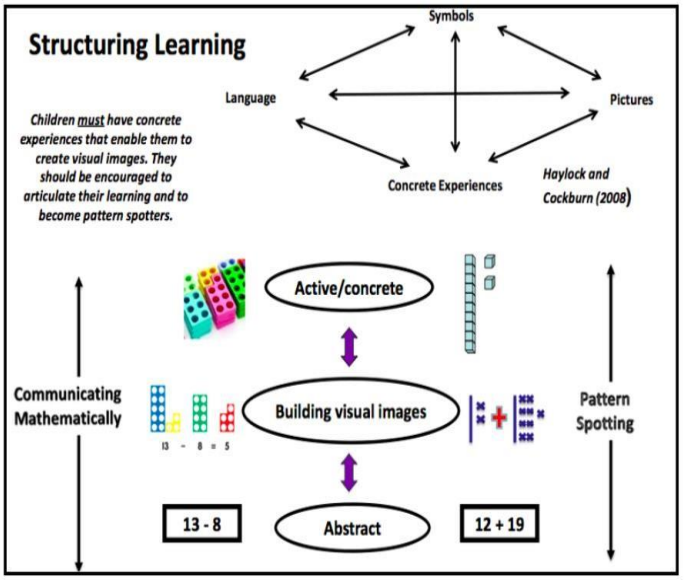
Children develop an understanding of a mathematical concept through the three steps (or representations) of concrete-pictorial-abstract approach.

Concrete representation - a pupil is first introduced to an idea or a skill by acting it out with real objects.

This is a 'hands on' component using real objects and it is the foundation for conceptual understanding.

Pictorial representation - a pupil has sufficiently understood the hands-on experiences performed and can now relate them to representations, such as a diagram or picture of the problem.

Abstract representation - a pupil is now capable of representing problems by using mathematical notation, for example: 12 ÷ 2 = 6



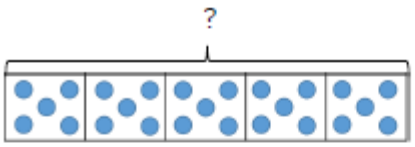
Possible models and images for addition and subtraction

The following models and images have been suggested by the White Rose Maths Calculation Policy They demonstrate the models and images which could be used to support the teaching of the different concepts.

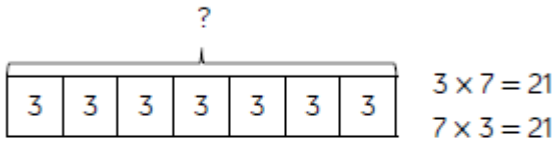
The benefits have been explained for each model or manipulative suggested.

Bar Model

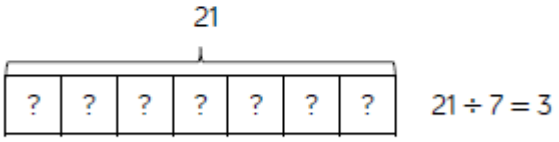
Children can use the single bar model to represent multiplication as repeated addition. They could use counters, cubes or dots within the bar model to support calculation before moving on to placing digits into the bar model to represent the multiplication.



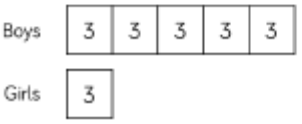
Division can be represented by showing the total of the bar model and then dividing the bar model into equal groups.
It is important when solving word problems that the bar model represents the problem.



Children may look at scaling problems. In the case opposite, more than one bar model is useful to represent this, e.g. There are 3 girls in a group. There are 5 times more boys than girls. How many boys are there?



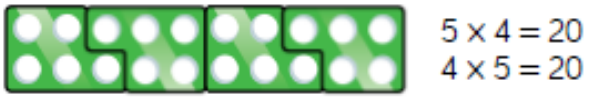
The bar model provides an opportunity to compare the groups.



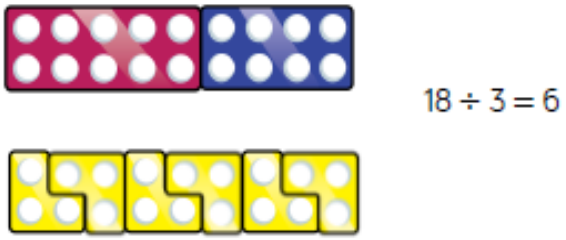
Numicon

Numicon supports children's understanding of multiplication as repeated addition.

Children can build multiplications in a row using the number shapes. They can then use the tens Numicon along with other necessary shapes over the top of the row to check the total. Using the Numicon in multiplication can support children in discovering patterns of multiplication



When dividing, Numicon supports children's understanding of division as grouping. Children make the number they are dividing and then place the Numicon shape they are dividing by over the top of the number to find how many groups of the number there are altogether e.g. There are 6 groups of 3 in 18.



Bead Strings

$5 \times 3 = 15$
$3 \times 5 = 15$

$15 \div 3 = 5$

$5 \times 3 = 15$
$3 \times 5 = 15$

$15 \div 5 = 3$

$4 \times 5 = 20$
$5 \times 4 = 20$

$20 \div 4 = 5$

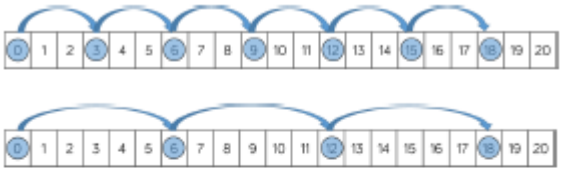
Bead strings to 100 can support children in their understanding of multiplication as repeated addition. Children can build the multiplication using the beads. The colour of beads supports children in seeing how many groups of 10 they have, to calculate the total more efficiently.

Encourage children to count in multiples as they build the number e.g. 4, 8, 12, 16, 20.

Children can also use the bead string to count forwards and backwards in multiples, moving the beads as they count.

When dividing, children build the number they are dividing and then group the beads into the number they are dividing by e.g. 20 divided by 4 – Make 20 and then group the beads into groups of four. Count how many groups you have made to find the answer.

Number Tracks



$6 \times 3 = 18$
$3 \times 6 = 18$

$18 \div 3 = 6$

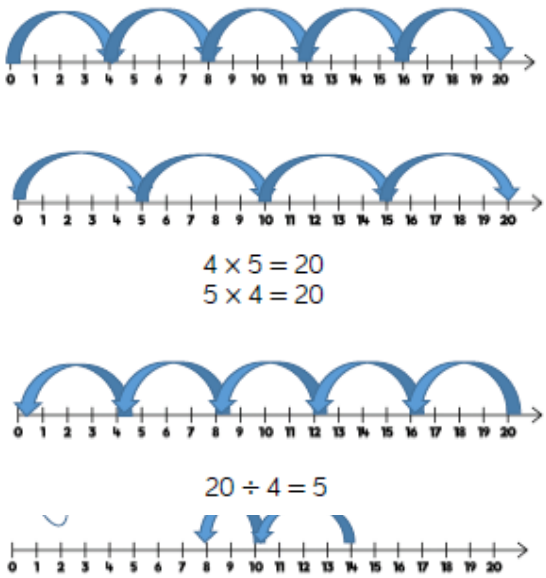
Number tracks are useful to support children to count in multiples, forwards and backwards. Moving counters or cubes along the number track can support children to keep track of their counting. Translucent counters help children to see the number they have landed on whilst counting.

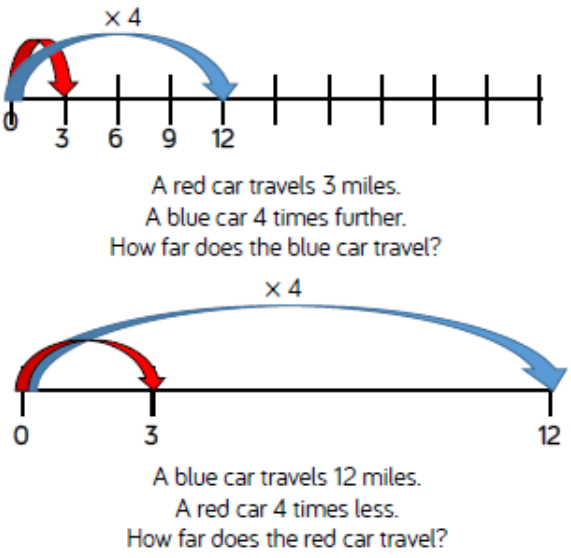
When multiplying, children place their counter on 0 to start and then count on to find the product of the numbers.

When dividing, children place their counter on the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0.

Children record how many jumps they have made to find the answer to the division. Number tracks can be useful with smaller multiples but when reaching larger numbers they can become less efficient.

Number Lines (labelled and blank)





Labelled number lines are useful to support children to count in multiples, forwards and backwards as well as calculating single-digit multiplications.

When multiplying, children start at 0 and then count on to find the product of the numbers.

When dividing, start at the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0. Children record how many jumps they have made to find the answer to the division.

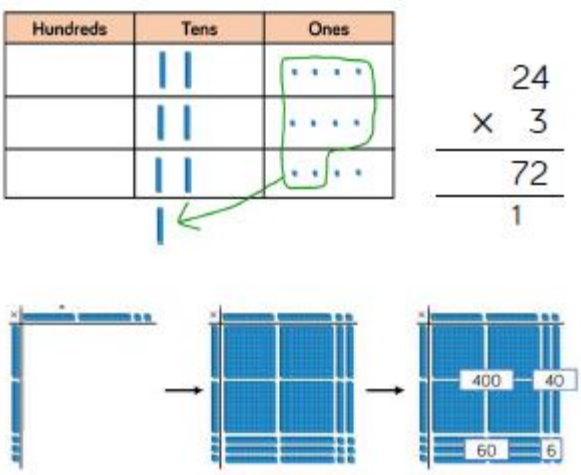
Labelled number lines can be useful with smaller multiples, however they become inefficient as numbers become larger due to the required size of the number line.

Children can use blank number lines to represent scaling as multiplication or division.

Blank number lines with intervals can support children to represent scaling accurately. Children can label intervals with multiples to calculate scaling problems.

Blank number lines without intervals can also be used for children to represent scaling.

Base 10 (Multiplication)



Using Base 10 is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written representations match.

As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 becomes less efficient due to the amount of equipment and number of exchanges needed.

Base 10 also supports the area model of multiplication well.

Children use the equipment to build the number in a rectangular shape which they then find the area of by calculating the total value of the pieces This area model can be linked to the grid method or the formal column method of multiplying 2-digits by 2-digits.

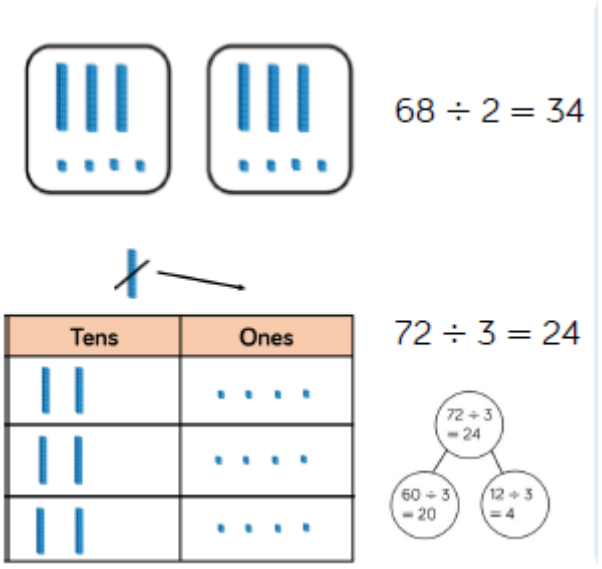
Base 10 (Division)

Using Base 10 is an effective way to support children's understanding of division.

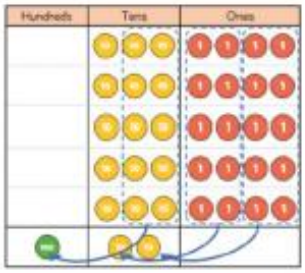
When numbers become larger, it can be an effective way to move children from representing numbers as ones towards representing them as tens and ones in order to divide. Children can then share the Base 10 between different groups e.g. by drawing circles or by rows on a place value grid.

When they are sharing, children start with the larger place value and work from left to right. If there are any left in a column, they exchange e.g. one ten for ten ones.

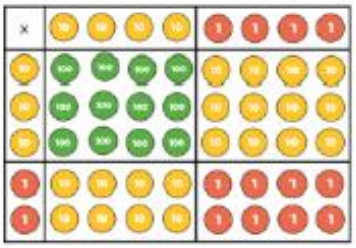
When recording, encourage children to use the part-whole model so they can consider how the number has been partitioned in order to divide. This will support them with mental methods.



Place Value counters (Multiplication)



$$\begin{array}{r} 34 \\ \times\ 5 \\ \hline 170 \\ \hline 1\ 2 \end{array}$$



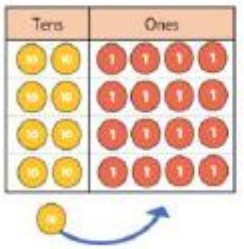
$$\begin{array}{r} 44 \\ \times\ 32 \\ \hline 8 \\ 80 \\ 120 \\ +\ 1200 \\ \hline 1408 \\ \hline 1 \end{array}$$

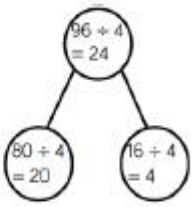
Using place value counters is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written match.

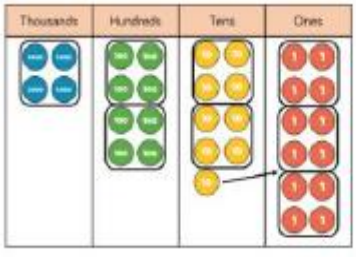
As numbers become larger in multiplication or the amounts of groups becomes higher, place value counters become less efficient due to the amount of equipment and number of exchanges needed The counters should be used to support the understanding of the written method rather than support the arithmetic.

Place value counters also support the area model of multiplication well. Children can see how to multiply 2-digitnumbersby 2-digitnumbers.

Place Value counters (Division)







$$\begin{array}{r} 1223 \\ 4\overline{)\,4892} \end{array}$$

Using place value counters is an effective way to support children's understanding of division.

When working with smaller numbers, children can use place value counters to share between groups. They start by sharing the larger place value column and work from left to right. If there are any counters left over once they have been shared, they exchange the counter e.g. exchange one ten for ten ones. This method can be linked to the part-whole model to support children to show their thinking.

Place value counters also support children's understanding of short division by grouping the counters rather than sharing them. Children work from left to right through the place value columns and group the counters in the number they are dividing by. If there are any counters left over after they have been grouped, they exchange the counter e.g. exchange one hundred for ten tens.