Elburton Primary School

Addition & Subtraction
Models & Representations Policy

October 2022	Policy Agreed
July 2024	Policy Review Date
Date	**Description**

This progression in calculations policy has been devised using the guidance from White Rose Maths Hub as well as the NCETM to support understanding in the expectations for fluency of the National Curriculum in England (2014) as well as the progression of calculation concepts through a child's mathematical development. Teaching staff have adapted and agreed on the guidance given in order to make it bespoke to the curriculum here at Elburton Primary School.

Principles

- To support children's development and understanding of calculations through the use of concrete, pictorial and abstract methods.
- To support children to develop a deeper understanding of number as well as mental and written calculation.
- To develop, using a Mastery Maths approach, number awareness and fluency, which is supported through the use of models and images.
- To ultimately develop proficiency with the expected formal written methods by the end of Year 6.
- Specific practical resources and models have been suggested as well as the benefits of these for manipulative to support children in developing the conceptual understanding that will enable them to move efficiently towards the formal written methods expected.
- It is expected that all children will work towards the fluency goals for each age group but that, where necessary, teachers will use approaches and materials from NCETM, White Rose Maths and earlier year groups to bridge any gaps in a child's understanding.
- Teachers should have an understanding of the expectations and progression for all year groups, regardless of which year group they teach.
- All teachers have progression maps linked to the objectives set out in the National Curriculum 2014.

Concrete-Pictorial-Abstract Representations

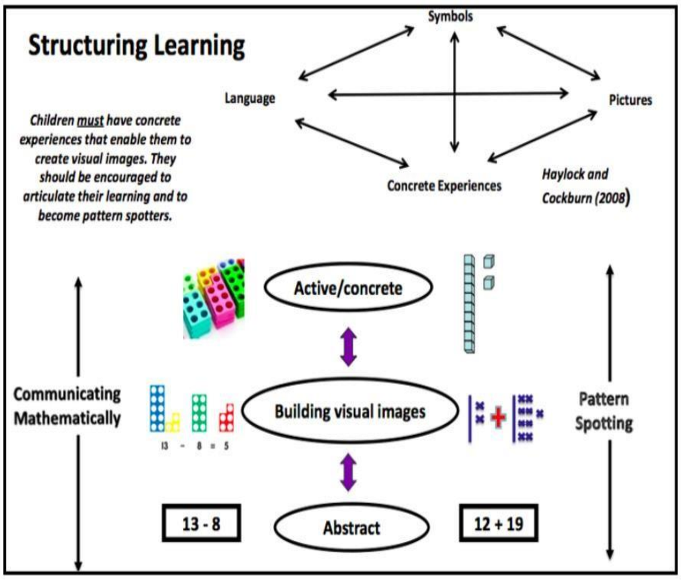
Children develop an understanding of a mathematical concept through the three steps (or representations) of concrete-pictorial-abstract approach.

Concrete representation - a pupil is first introduced to an idea or a skill by acting it out with real objects.

This is a 'hands on' component using real objects and it is the foundation for conceptual understanding.

Pictorial representation - a pupil has sufficiently understood the hands-on experiences performed and can now relate them to representations, such as a diagram or picture of the problem.

Abstract representation - a pupil is now capable of representing problems by using mathematical notation, for example: 12 ÷ 2 = 6



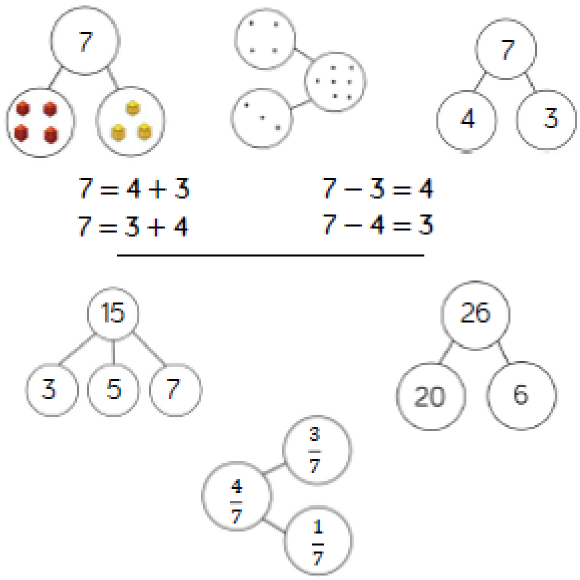
Possible models and images for addition and subtraction

The following models and images have been suggested by the White Rose Maths Calculation Policy They demonstrate the models and images which could be used to support the teaching of the different concepts.

The benefits have been explained for each model or manipulative suggested.

Part Whole Model

This **part-whole model** supports children in their understanding of aggregation and partitioning.



When the parts are complete and the whole is empty, children use aggregation to add the parts together to find the total.

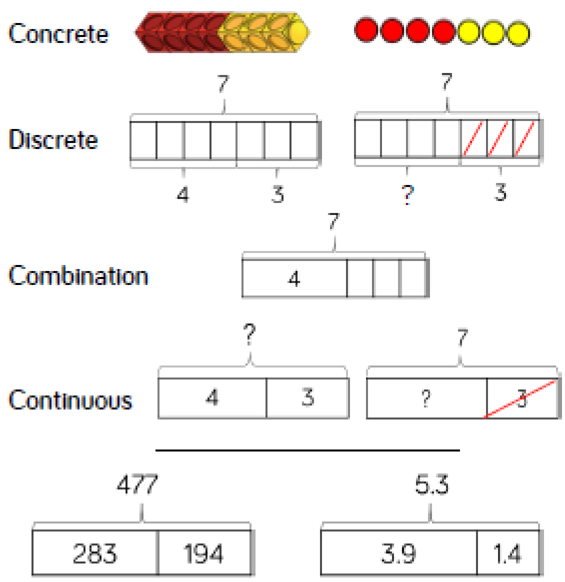
When the whole is complete and at least one of the parts is empty, children use partitioning to find the missing part.

Part-whole models can be used to partition a number into two or more parts, or to help children to partition a number into tens and ones or other place value columns.

In Key Stage 2, children can apply their understanding of the part-whole model to add and subtract fractions, decimals and percentages.

Single Bar Model

The **single bar model** is another type of a part-whole model that can support children in representing calculations to help them unpick the structure.



Cubes and counters can be used in a line as a concrete representation of the bar model.

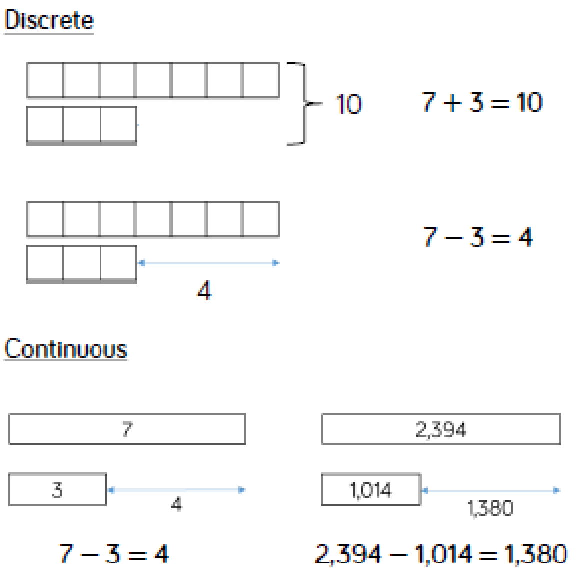
Discrete bar models are a good starting point with smaller numbers. Each box represents one whole.

The combination bar model can support children to calculate by counting on from the larger number.

Continuous bar models are useful for a range of values. Each rectangle represents a number. The question mark indicates the value to be found.

In Key Stage 2, children can use bar models to represent larger numbers, decimals and fractions.

Multiple Bar Model



The **multiple bar model** is a good way to compare quantities whilst still unpicking the structure.

Two or more bars can be drawn, with a bracket labelling the whole positioned on the right hand side of the bars. Smaller numbers can be represented with a discrete bar model whilst continuous bar models are more effective for larger numbers.

Multiple bar models can also be used to represent the difference in subtraction. An arrow can be used to model the difference.

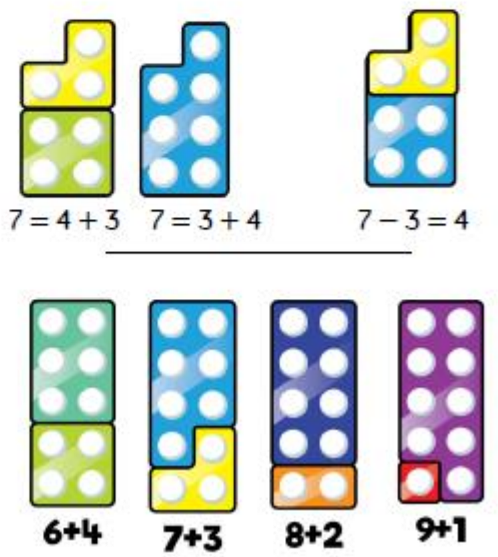
Numicon

Numicon shapes can be useful to support children to subitise numbers as well as explore aggregation, partitioning and number bonds.

When adding numbers, children can see how the parts come together making a whole. As children use Numicon more often, they can start to subitise the total due to their familiarity with the shape of each number.

When subtracting numbers, children can start with the whole and then place one of the parts on top of the whole to see what part is missing.

Children can also work systematically to find number bonds. As they increase one number by 1, they can see that the other number decreases by 1 to find all the possible number bonds for a number.



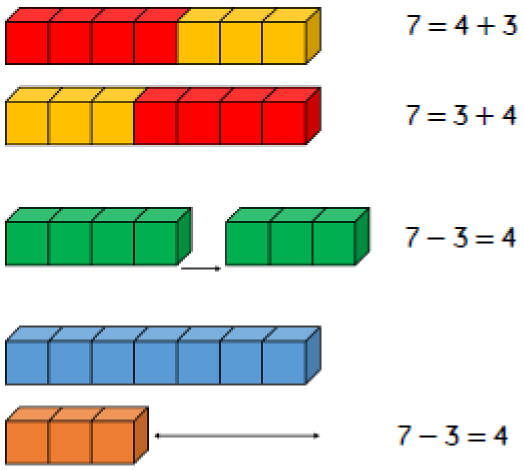
Multilink cubes

Multilink cubes can be useful to support children with the addition and subtraction of one-digit numbers.

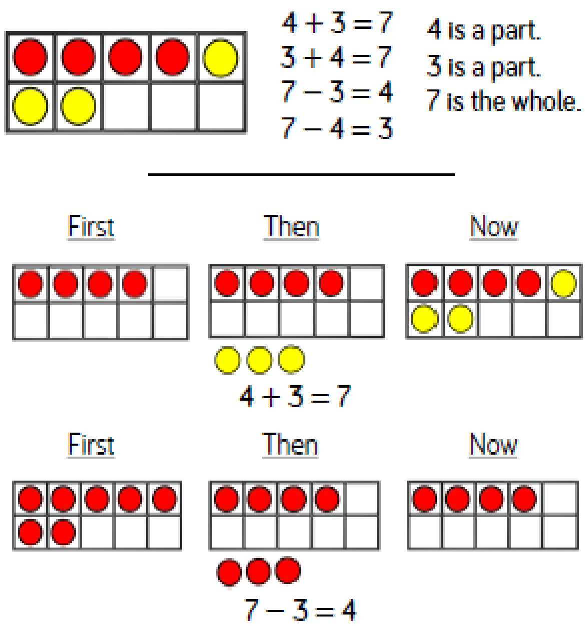
When adding numbers, children can see how the parts come together to make a whole. Children could use two different colours of cubes to represent the numbers before putting them together to create the whole.

When subtracting numbers, children can start with the whole and then remove the number of cubes that they are subtracting in order to find the answer. This model of subtraction is reduction, or take away.

Cubes can also be useful to look at subtraction as difference. Here, both numbers are made and then lined up to find the difference between the numbers.



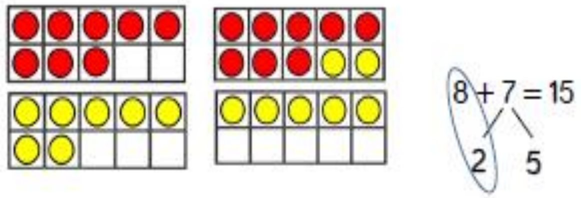
Tens Frames

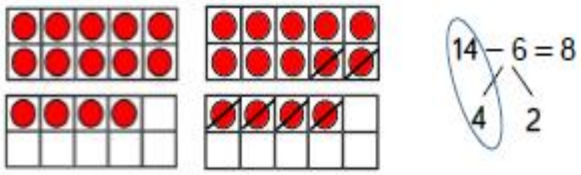


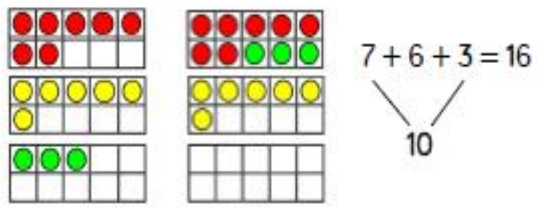
When adding and subtracting within 10, the ten frame can support children to understand the different structures of addition and subtraction. Using the language of parts and wholes represented by objects on the ten frame introduces children to aggregation and partitioning.

Using these structures, the ten frame can enable children to find all the number bonds for a number. Children can also use ten frames to look at augmentation (increasing a number) and take-away (decreasing a number). This can be introduced through a first, then, now structure which shows the change in the number in the 'then' stage. This can be put into a story structure to help children understand the change.

When adding two single digits, children can make each number on separate ten frames before moving part of one number to make 10 on one of the ten frames. This supports children to see how they have partitioned one of the numbers to make 10, and makes links to effective mental methods of addition.



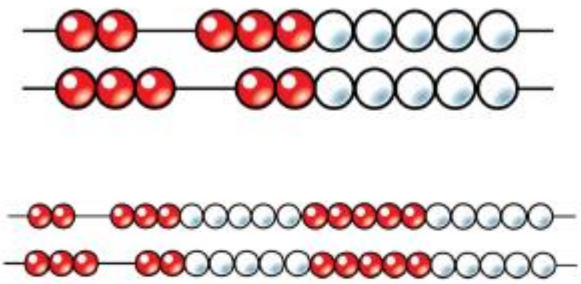




When subtracting a one-digit number from a two-digit number, firstly make the larger number on 2 ten frames. Remove the smaller number, thinking carefully about how you have partitioned the number to make 10, this supports mental methods of subtraction.

When adding three single-digit numbers, children can make each number on 3 separate 10 frames before considering which order to add the numbers in. They may be able to find a number bond to 10 which makes the calculation easier. Once again, the ten frames support the link to effective mental methods of addition as well as the importance of commutativity.

<u>Bead Strings and Rekenreks</u>



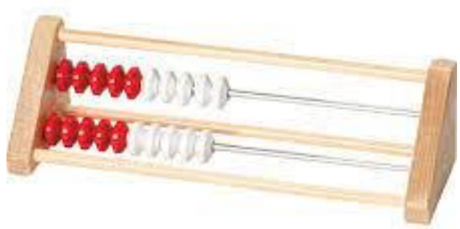
Different sizes of bead strings can support children at different stages of addition and subtraction.

Bead strings to 10 are very effective at helping children to investigate number bonds up to 10. They can help children to systematically find all the number bonds to 10 by moving one bead at a time to see the different numbers they have partitioned the 10 beads into e.g. 2 +8 =10, move one bead, 3 +7 =10.

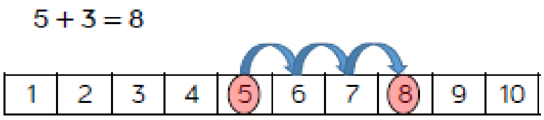
Bead strings to 20 work in a similar way but they also group the beads in fives. Children can apply their knowledge of number bonds to 10 and see the links to number bonds to 20.

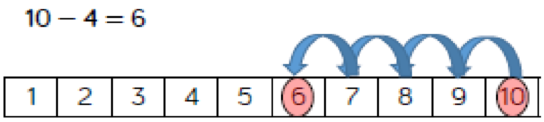
Bead strings to 100 are grouped in tens and can support children in number bonds to 100 as well as helping when adding by making ten. Bead strings can show a link to adding to the next 10 on number lines which supports a mental method of addition.

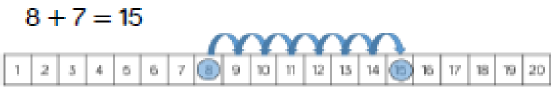
Rekenreks support our use of the NCETM Mastering Number project. In Key Stage 1, these are used to develop subitising skills, doubling and as well as addition and subtraction understanding.



Number Tracks







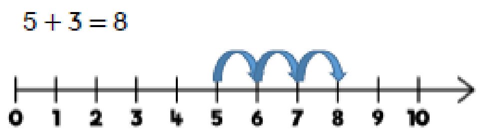
Number tracks are useful to support children in their understanding of augmentation and reduction.

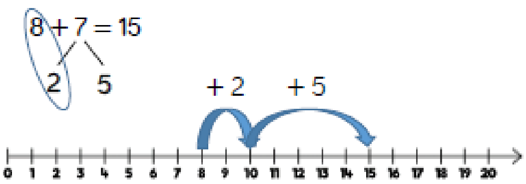
When adding, children count on to find the total of the numbers. On a number track, children can place a counter on the starting number and then count on to find the total.

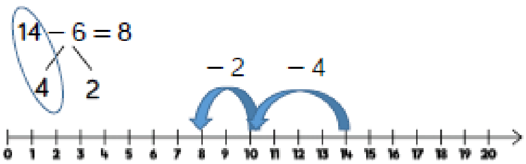
When subtracting, children count back to find their answer. They start at the minuend and then take away the subtrahend to find the difference between the numbers.

Playing board games can help children to become familiar with the idea of counting on using a number track before they move on to number lines.

Number Lines (labelled and blank)







Labelled number lines support children in their understanding of addition and subtraction as augmentation and reduction.

Children can start by counting on or back in ones, up or down the number line. This skill links directly to the use of the number track.

Progressing further, children can add numbers by jumping to the nearest 10 and then jumping to the total. This links to the making 10 method which can also be supported by ten frames. The smaller number is partitioned to support children to make a number bond to 10 and to then add on the remaining part.

Children can subtract numbers by firstly jumping to the nearest 10. Again, this can be supported by ten frames so children can see how they partition the smaller number into the two separate jumps.

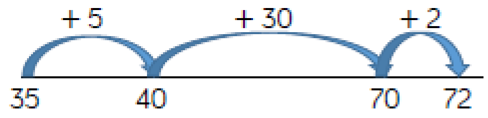
Blank number lines provide children with a structure to add and subtract numbers in smaller parts.

Developing from labelled number lines, children can add by jumping to the nearest 10 and then adding the rest of the number either as a whole or by adding the tens and ones separately.

Children may also count back on a number line to subtract, again by jumping to the nearest 10 and then subtracting the rest of the number.

Blank number lines can also be used effectively to help children subtract by finding the difference between numbers. This can be done by starting with the smaller number and then counting on to the larger number. They then add up the parts they have counted on to find the difference between the numbers.

$35 + 37 = 72$



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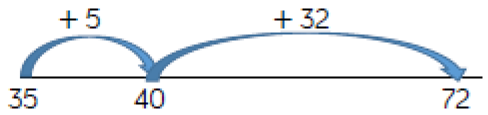
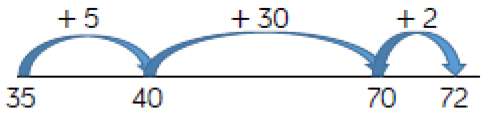


$72 - 35 = 37$



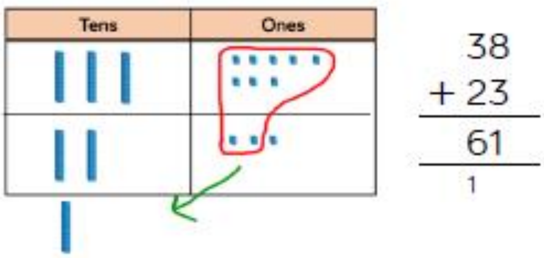
Base 10 (+ and -)
Addition

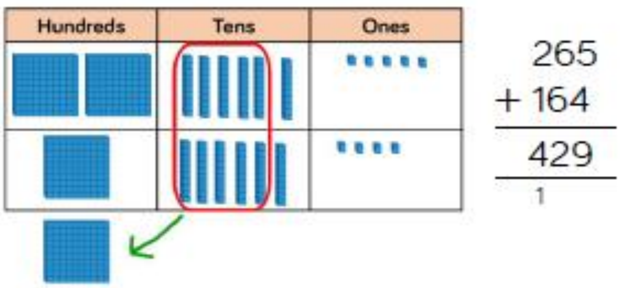
Using Base 10 is an effective way to support children's understanding of column addition and subtraction. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

Children should first add or subtract without an exchange before moving on to addition or subtraction with exchange.

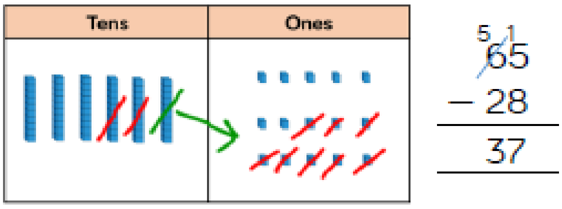
For addition:

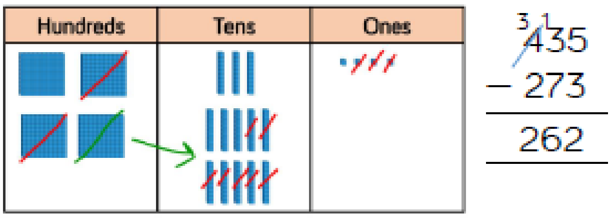
When adding, always start with the smallest place value column. Here are some questions to support children.





Subtraction





For subtraction:

When building the model, children should just make the minuend using Base 10, they then subtract the subtrahend. Highlight this difference to addition to avoid errors by making both numbers.

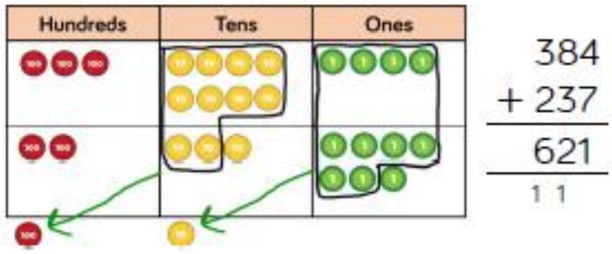
Children start with the smallest place value column.

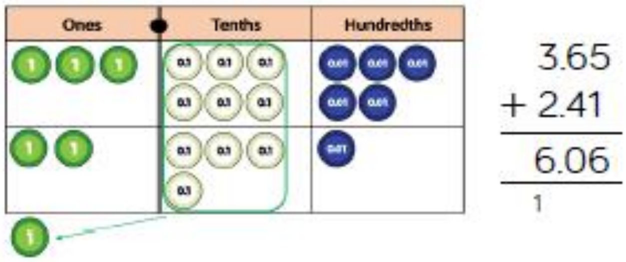
When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.

This model is efficient with up to 4-digit numbers.

Place Value counters (+ and -)

Addition



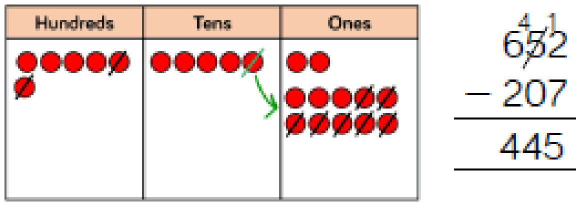


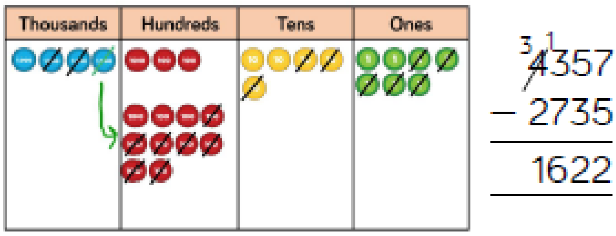
Using place value counters is an effective way to support children's understanding of column addition and subtraction. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

For addition:

Children should first add without an exchange before moving on to addition with exchange. Different place value counters can be used to represent larger numbers or decimals. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns. When adding money, children can also use coins to support their understanding. It is important that children consider how the coins link to the written calculation especially when adding decimal amounts.

Subtraction





For subtraction:

Children should first subtract without an exchange before moving on to subtraction with exchange. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

When building the model, children should just make the minuend using counters, they then subtract the subtrahend. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.